

Viscosities of the quasigluon plasma

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Abstract. We investigate bulk and shear viscosities of the gluon plasma within relaxation time approximation to an effective Boltzmann-Vlasov type kinetic theory by viewing the plasma as describable in terms of quasigluon excitations with temperature dependent self-energies. The found temperature dependence of the transport coefficients agrees fairly well with available lattice QCD results. The impact of some details in the quasigluon dispersion relation on the specific shear viscosity is discussed.

1. Introduction

Bulk ζ and shear η viscosities are important properties of strongly interacting matter. Their firm knowledge is a necessary prerequisite for dynamical descriptions of relativistic heavy-ion collisions, but also in cosmology and astrophysics. The success of ideal relativistic hydrodynamic simulations, cf. e. g. [1], in quantitatively describing the collective flow behaviour of the matter produced at the Relativistic Heavy Ion Collider led to the conclusion that at most small dissipative effects are present in the created medium [2, 3, 4]. Later investigations by means of dissipative relativistic hydrodynamics, cf. e. g. [5, 6, 7, 8], confirmed this observation.

However, the quantum mechanical uncertainty principle enforces a fundamental lower bound on the shear viscosity [9] for any physical system. Moreover, unitarity arguments led to a lower limit conjecture for the specific shear viscosity [10, 11] $(\eta/s)_{KSS} \geq \hbar/(4\pi)$ (Kovtun-Son-Starinets bound, s is the entropy density). The bulk viscosity, in contrast, is exactly zero in conformal theories. In QCD, however, it is only (approximately) zero at large temperatures T , while for T close to the deconfinement transition temperature T_c the specific bulk viscosity ζ/s is expected to become large [12, 13, 14].

The viscosity coefficients were recently evaluated by means of non-perturbative first-principle numerical lattice QCD calculations [15, 16, 17, 18]. Other analytic approaches are based on the Kubo-formalism [19] or on linearized Boltzmann kinetic theory [19, 20, 21, 22]. In fact, it was shown that in scalar theory [19, 20] and in hot QED [21, 22] both approaches are equivalent. For small QCD running coupling, perturbative QCD (pQCD) kinetic theory results at leading order for shear and bulk viscosities were reported in [23, 24] and [25], respectively.

2. Bulk and shear viscosity coefficients

We calculate bulk and shear viscosities of the gluon plasma by assuming that it can be described by quasigluon excitations with T -dependent self-energy $\Pi(T)$, cf. [26, 27]. In local thermal equilibrium, this picture is comprised within a quasiparticle model (QPM), where the gluon dispersion relation reads $E^0 = \sqrt{\vec{p}^2 + \Pi(T)}$. $\Pi(T) = T^2 G^2(T)/2$ contains the effective coupling $G^2(T) = 16\pi^2 / \left(11 \ln [\lambda(T - T_s)/T_c]^2 \right)$ with parameters λ and T_s . The QPM works impressively well for describing equilibrium lattice QCD thermodynamics, cf. [28, 29, 30].

By means of an effective kinetic theory [19, 20], the model can be extended to systems facing small deviations from local thermal equilibrium. In fact, as T is space-time dependent in general, so is the dispersion relation $E(x)$. The corresponding energy-momentum tensor reads $T^{\mu\nu}(x) = \int \frac{d^3\vec{p}}{(2\pi)^3 E(x)} p^\mu(x) p^\nu(x) b(x, p) + g^{\mu\nu} B(\Pi(x))$, where $b(x, p)$ represents the quasigluon distribution function entering $E(x)$ and $p^\alpha = (E, \vec{p})$. This expression for $T^{\mu\nu}$ is interwoven with a Boltzmann-Vlasov type kinetic equation [19]. The mean field term $B(\Pi)$ is necessary for satisfying locally energy and momentum conservation, $\partial_\mu T^{\mu\nu} = 0$, established when [26, 27] $\partial B / \partial \Pi = -\frac{1}{2} \int \frac{d^3\vec{p}}{(2\pi)^3 E(x)} b(x, p)$. This also guarantees thermodynamic self-consistency of the approach in thermal equilibrium and $E = \delta T^{00} / \delta b$ as a basic principle of statistical mechanics.

Expanding $b(x, p) = b^0(x, p) + \delta b(x, p)$ for small disturbances δb from equilibrium b^0 , $T^{\mu\nu}$ is decomposed in an equilibrium part and first-order corrections thereof. Using the relaxation time approximation for determining δb in the latter, one reads off [27] for η and ζ in the local fluid rest frame by comparison with the phenomenological definitions [31]

$$\eta = \frac{1}{15T} \int \frac{d^3p}{(2\pi)^3} n(T) [1 + d^{-1} n(T)] \frac{\tau}{(E^0)^2} \vec{p}^4, \quad (1)$$

$$\zeta = \frac{1}{T} \int \frac{d^3p}{(2\pi)^3} n(T) [1 + d^{-1} n(T)] \frac{\tau}{(E^0)^2} \left\{ \left[(E^0)^2 - T^2 \frac{\partial \Pi}{\partial T^2} \right] \frac{\partial P}{\partial \epsilon} - \frac{1}{3} \vec{p}^2 \right\}^2, \quad (2)$$

where $n(T) = d(e^{E^0/T} - 1)^{-1}$ with degeneracy factor $d = 16$, $\partial P / \partial \epsilon$ is the squared speed of sound and τ the relaxation time. These results generalize previous work [32, 33], in which excitations with constant mass M were considered. While the expression for η looks formally the same, cf. also [34], ζ in Eq. (2) contains a combination of E^0 and self-energy derivative in contrast to [32, 33, 34]. It is this particular combination which guarantees $\zeta \rightarrow 0$ in the conformal limit.

3. Numerical results

For numerically evaluating Eqs. (1) and (2), we employ as ansatz for the relaxation time $\tau = [a_1 T G^4(T) \ln(a_2 / G^2(T))]^{-1}$, which is inspired by considerations in [35]. It turns out that with this ansatz for τ , both η and ζ resemble at large T the behaviour with temperature and coupling known from pQCD [23, 24, 25]. The parameters in $G^2(T)$ are adjusted such that thermal equilibrium lattice QCD results for s/T^3 from [36, 37] are described by the QPM. Local thermal equilibrium quantities, such as s , are obtained by inserting $n(T)$ for $b^0(x, p)$ into $T^{\mu\nu}(x)$, cf. [27]. Furthermore, we fit the parameters in τ as $a_1 = 2.587 \times 10^{-2}$ and $a_2 = (\mu_*/T)^2$ (Fit 1) or as $a_1 = 3.85 \times 10^{-2}$ and $a_2 = 2(\mu_*/T)^2$ (Fit 2), where $\mu_*/T = 2.765$ as in pQCD [24]. QPM results for η/s and the corresponding relaxation times are exhibited in Fig. 1, while results for ζ/s are depicted in Fig. 2.

Good agreement with available lattice QCD results is found. In particular, η/s exhibits a minimum in the vicinity of T_c , which is common for a variety of liquids and gases [11, 40]. We find $(\eta/s)_{min}^* = 0.096$ at $T = 1.22 T_c$ for Fit 2. The exact location of the minimum is driven by a_2 in τ . With increasing T , η/s increases rather mildly, while for $T \rightarrow T_c^+$ it dramatically rises in line with the relaxation time and the phenomenon of critical slowing down.

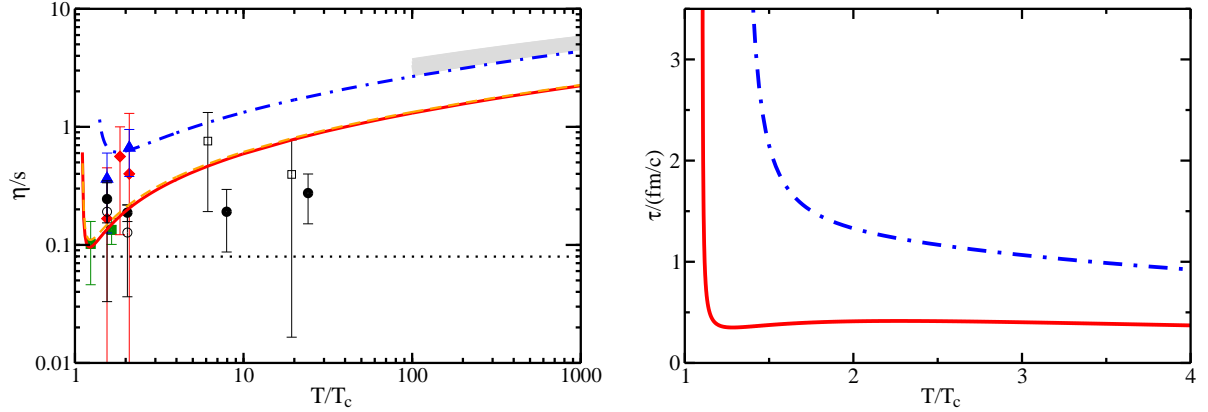


Figure 1. Left: Specific shear viscosity as a function of T/T_c . Dash-dotted and solid curves exhibit QPM results for Fit 1 and Fit 2, respectively, compared with lattice QCD data from [17] (full squares), [15] (diamonds and triangles) and [16] (open squares, open and full circles). The dotted line exhibits the KSS bound. The grey band at large T depicts the pQCD result η_{NLL}/\tilde{s} , where η_{NLL} is from [24] and \tilde{s} is from [38]. The dashed curve (almost on top of the solid curve) shows η/s for Fit 2 for constant mass $M = 200$ MeV instead of $\Pi(T)$ in E^0 . Right: Corresponding relaxation times τ . The behaviour is crucially determined by the parameter a_2 .

For comparison, η/s for Fit 2 based on Eq. (1) but for particles with constant T -independent mass $M = 200$ MeV is also exhibited in Fig. 1 (dashed curve). Only the region around the minimum in η/s is affected (the minimal value is about 14% larger than $(\eta/s)_{min}^*$), while at large T details in E^0 become irrelevant. Increasing M generally leads to a decrease in η/s . For $M = 1$ GeV a minimal value 9% smaller than $(\eta/s)_{min}^*$ is found.

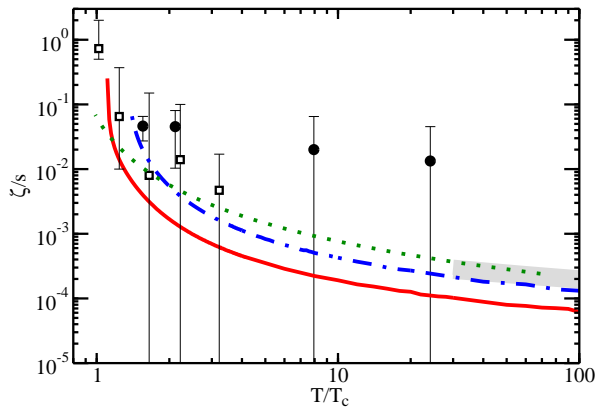


Figure 2. Specific bulk viscosity as a function of T/T_c . Dash-dotted and solid curves exhibit QPM results for Fit 1 and Fit 2, respectively, compared with available lattice QCD data from [18] (squares) and [16] (circles). The dotted curve shows for comparison results from holographic QCD [39]. The grey band at large T depicts the pQCD result ζ_{LO}/\tilde{s} , where ζ_{LO} is from [25] and \tilde{s} is from [38].

4. Conclusion

We derived expressions for bulk and shear viscosities within relaxation time approximation to an effective kinetic theory for the gluon plasma assumed to be composed of quasigluon excitations with T -dependent dispersion relation E^0 . Medium effects show up in η only implicitly via E^0 , while they are prominently evident in ζ . Specifying the relaxation time and adjusting model parameters to thermal equilibrium lattice QCD results, we find good agreement with available lattice QCD data for η/s and ζ/s for T close to T_c . The influence of moderate constant masses M on η/s is observed to be small and focussed on the region around its minimum. At large T our results resemble the parametric dependence on T and coupling known from pQCD.

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